Designing driver Hamiltonians for continuous-time quantum walks: background

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Our research investigated possible improvements to a quantum algorithm called a continuous-time quantum walk. We applied the quantum algorithm to a combinatorial optimisation problem called max-cut. In this poster we provide the background to our project.

What is quantum mechanics?

Quantum Mechanics describes the physics of atoms, we used it to solve problems through probabilities and vectors.

There are **4** rules to quantum mechanics:

Rule 1:
Any closed-system can be described by a state vector. State vectors have length one.

Vectors can be added, subtracted and multiplied by a number. To find the length – add the square of all the numbers and take the square root

A closed system is when there are no inputs or outputs.

The vector $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

has length $=\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}=1$

Rule 2:
Each number (also known as an amplitude) corresponds to the square-root of the probability of finding the system in a classical outcome.
A system is made up two coins.
0=Heads and 1=Tails
The state vector is:

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

Outcome	Probability
00 (2 heads)	½
01 (head and tail)	0
10 (tail and head)	0
11 (2 tails)	½

Rule 3:

The Schrödinger equation:

Tells you how to get from one state-vector to another state-vector.

What goes into the Schrödinger equation?

- The initial state vector
- Time
- The Hamiltonian (describes the energy)

Source: Jie Qi, Schrodinger's Cat, Flickr CC BY 2.0

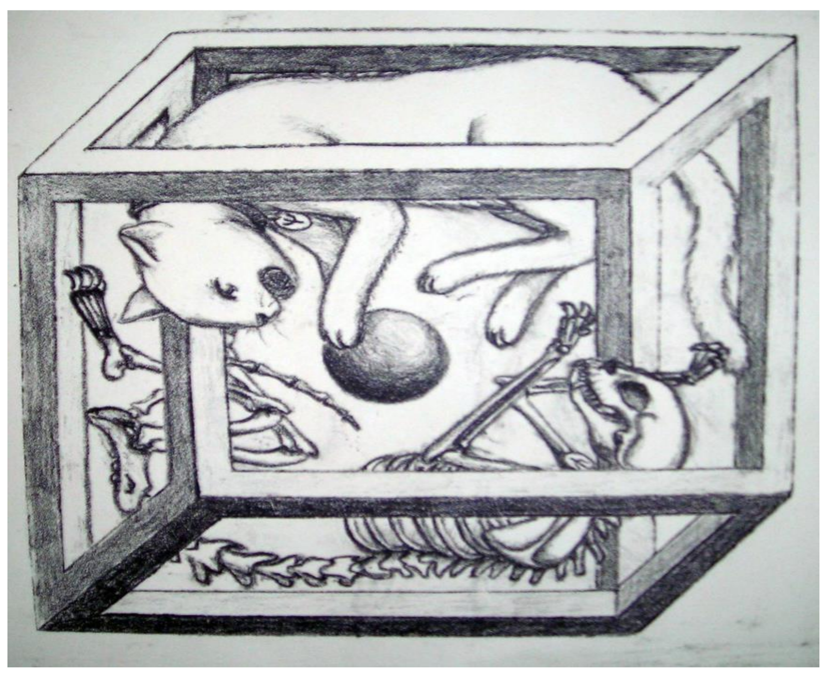


Figure 1: a cartoon of Schrödingers cat.

Rule 4:

Bigger systems need bigger vectors

Source: Classical Numismatic Group, Inc, Wikipedia, CC BY 2.5

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

What is max-cut?

If you can count the number of solutions to a problem, you are dealing with a **combinatorial optimisation problem**.

Max-cut is an example of a *combinatorial optimisation problem*.

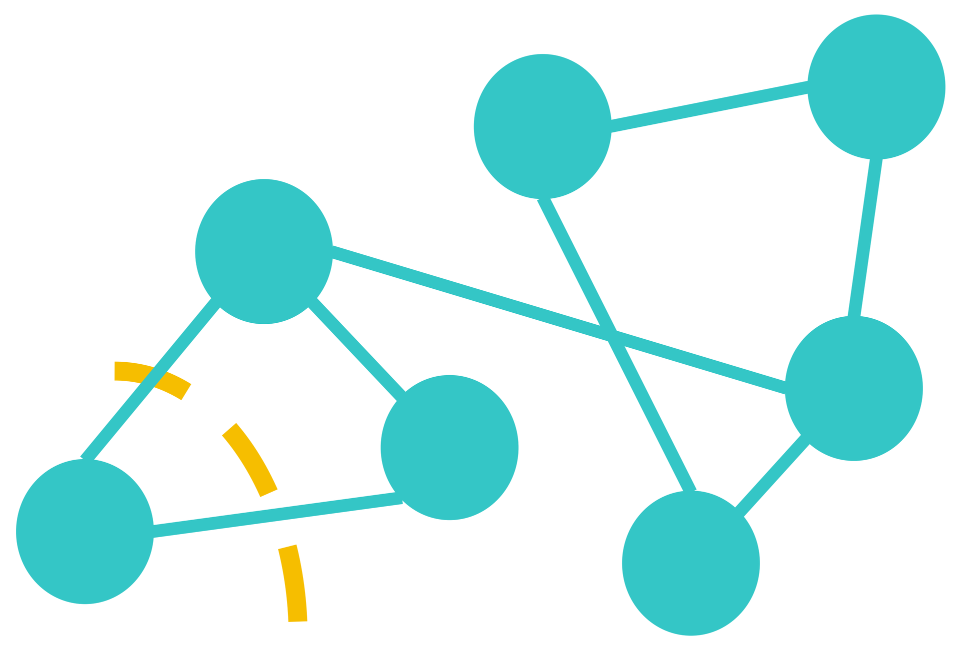


Figure 2: an example graph with a cut shown in yellow. Can you find the longest cut?

Figure 2 shows a graph made up of nodes (the circles) and edges (the lines between nodes).

The length of the cut shown in yellow in Figure 2 is 2 because it cuts through 2 nodes.

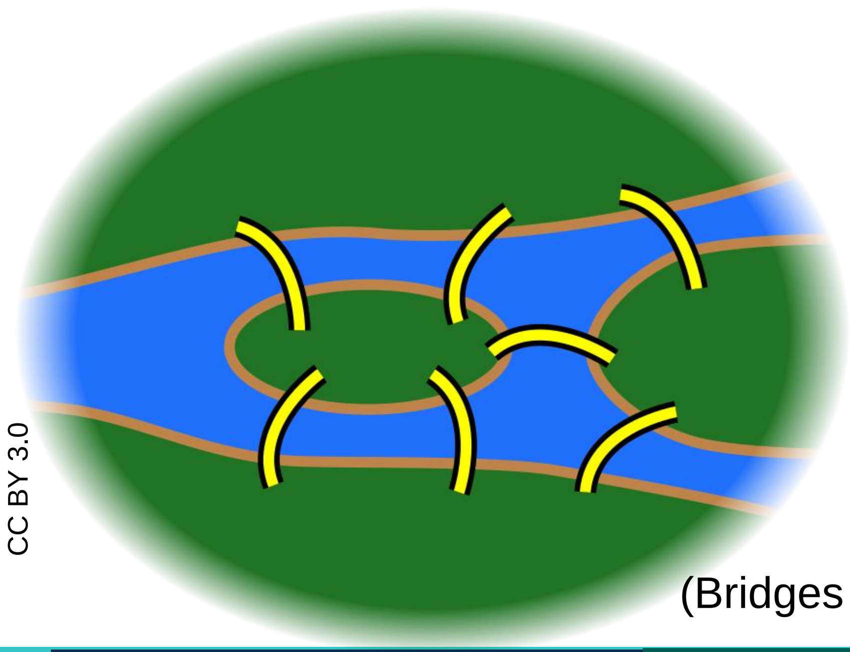
Length of the cut = number of edges the cut goes through.

In max-cut the aim is to find the longest cut in a graph. There can be a lot of cuts to try making the problem difficult.

A brain-teaser!
Max-cut is a problem from graph theory. An early problem was the Seven Bridges of Königsberg.

Can you find a way to cross every bridge once in the picture below?

Source: Chris-Mar-tin, Wikipedia, CC BY 3.0



(Bridges are in yellow)